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GABA CAMPUS - ELDORET

MAIN EXAMINATION

JANUARY– APRIL 2023 TRIMESTER

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS AND ACTUARIAL SCIENCE

REGULAR PROGRAMME

MAT 335: METHODS OF APPLIED MATHEMATICS I

DATE: April 2023	**Duration: 2 Hours**
Instructions: Answer Question ONE and any other TWO Questions	

Q1.

a) The displacement of a vibrating string is described by the equation

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

With the boundary conditions

$x = 0$, $u(0,t) = 0$
$x = l$, $u(l,t) = 0$
And the initial conditions: $t = 0$, $u(x,0) = \phi(x)$

Apply the method of separation of variables to find the general solution.

(10 Marks)

b) Find the general solution of the first order differential equation by use of transforms

$\frac{dx}{dt} - 2x = 4$ given that $x(0) = 1$ **(6 Marks)**

c) Find the value of $\Gamma(\frac{5}{2})$ **(4 Marks)**

d) Use Gamma function to evaluate $\int_0^\infty x^3 e^{-4x} dx$ **(5 Marks)**

e) Use Beta function to evaluate $\int_0^1 x^5 (1-x)^4 dx$ **(5 Marks)**

Q2.

a) Prove that $L\{\cosh at\} = \frac{s}{s^2 - a^2}$ **(3 Marks)**

b) Solve the boundary value problem

$$\frac{d^2x}{dt^2} - 3\frac{dy}{dx} + 2x = 2e^{3t}$$
$$x_0 = x(0) = 5$$
$$x_1 = \frac{d}{dx}\left(x(0)\right) = 7$$
(10 Marks)

c) Prove that $\Gamma(\tfrac{1}{2}) = \sqrt{\pi}$ **(7 Marks)**

Q3.

a) Solve the boundary value problem for the Laplace equation

$$U_{xx} + U_{yy} = 0$$

Where $U(x, y)$ represents the velocity potential of fluid particle in a certain domain, particularly inside a unit circle $x^2 + y^2 > 1$. **(12 Marks)**

b) Prove that $J_{-\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}}\cos x$ **(4 Marks)**

c) Find $L^{-1}\left\{\frac{5s+1}{s^2 - s - 12}\right\}$ **(4 Marks)**

Q4.

a) Apply L'Hospital Rule to calculate the Laplace transform

$$L\left\{\frac{\sin at}{t}\right\}$$
(5 Marks)

b) Given a Beta function $\beta(m,n) = \int x^{m-1}(1-x)^{n-1}\,dx$, prove that $\beta(m,n) = \beta(n,m)$

(3 Marks)

Find the Fourier series for the function

$$f(x) = \begin{cases} -x; & -\pi < x < 0 \\ 0; & 0 < x < \pi \\ f(x) & = f(x + 2\pi) \end{cases}$$
(12 Marks)

END