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MAIN EXAMINATION

SEPTEMBER - DECEMBER 2022

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS AND ACTUARIAL SCIENCE

REGULAR PROGRAMME

MAT 433: PARTIAL DIFFERENTIAL EQUATIONS II

DATE: DECEMBER 2022	Duration: 2 Hours
INSTRUCTIONS: Answer Question ONE and any other TWO Questions	

Q1.

a) State the second order linear partial differential equation (P.D.E.) in its general form. Hence name the three classical P.D.Es that are the canonical representations of this general 2nd order linear P.D.E. **(5 Marks)**

b) Given that $u = u(x, y)$, apply the change of variables$\xi = \xi(x,y))$,$\eta = \eta(x, y)$ to express the second order derivatives $u_x$and u_{yy} in terms of the variables ξ, η. **(7 Marks)**

c) Write the Fourier series expansion of a function $f(x)$ over $[-L, L]$. **(3 Marks)**

d) Solve using separation of variables $\frac{du}{dx} = 2\frac{du}{dt} + u$ where $u(x, 0) = 4e^{-3x}$ **(10 Marks)**

e) Find the characteristic equation of $2u_{xx} - 2u_{xy} + 5u_{yy} + 4 = 0$ **(5 Marks)**

Q2

a) With examples differentiate between linear, quasi linear and nonlinear PDE **(6 Marks)**

b) Solve the following initial boundary value problem for temperature in a homogeneous isotropic rod with insulated sides and no internal heat generation:

$$u_t = ku_{xx}, 0 < x < \pi, t > 0$$

$$u_x(0, t) = u_x(\pi, t) = 0$$

$$u(x, 0) = x \qquad 0 < x < \pi$$

to obtain the particular solution using separation of variables method. The given boundary conditions imply that both ends of the rod are also insulated. **(14 Marks)**

Q3

a) Find the characteristics of the equation $u_{xx} + u_{xy} - 2u_{yy} - 3u_x - 6u_y = 9(2x - y)$ and reduce it to the appropriate canonical form. **(10 Marks)**

b) A string is stretch between fixed points (0,0) and (1,0) and released at rest from the position $y = asin\pi x$ where A is a constant. Find the expression for its subsequent displacement y(0,t) and verify the results fully. Sketch the position of string at various instances of time.

(10 Marks)

Q4

a) Separate the variables and find its general solution $u_{xx} + 3u = u_t$ **(9 Marks)**

b) Classify the partial differential equation $xu_{xx} - xu_{yy} - 2u_x = 0$ and transform the P.D.E. to its canonical form. **(11 Marks)**

END