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MAIN EXAMINATION

SEPTEMBER - DECEMBER 2022

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS AND ACTUARIAL SCIENCE

REGULAR PROGRAMME

MAT 330: ORDINARY DIFFERENTIAL EQUATIONS II

DATE: DECEMBER 2022	Duration: 2 Hours
INSTRUCTIONS: Answer Question ONE and any other TWO Questions	

Q1.

a) i) Show that $y_1 = e^x \sin x$ and $y_2 = e^x \cos x$ are the linearly independent solutions of the differential equation $y'' - 2y' + 2y = 0$. **(3 Marks)**

ii) What is the general solution? **(1 Mark)**

iii) Find the general solution $y(x)$ with the initial conditions $y(0) = 2,\ y'(0) = 3$. **(3 Marks)**

b) By variation of parameters, find the general solution of the differential equation

$y'' - 4y' - 12y = 3e^{5t}$ **(8 marks)**

c) i) Apply variation of parameters to prove that for any given differential equation $y''+p(x)y'+q(x)y = r(x)$, the particular integral is given by

$$y_p = -y_1\int \frac{y_2 r}{W}dx + y_2\int \frac{y_1 r}{W}dx$$ **(8 mark)**

ii) Apply the relation in (i) above to find the general solution of the differential equation

$y''-2y'+y = e^x \log x$ **(7 marks)**

Q2

a) Show that the following differential equation is exact, hence solve the equation by reducing its order

$(1 + x^2)y'' + 4xy' + 2y = sec^2 x$ **(10 marks)**

b) Solve the differential equation below by first transforming it into normal form

$(y'' + y)\cot x + 2(y' + ytan\ x) = \sec x$ **(10 marks)**

Q3

a) Write the following equations in Sturm-Liouville form:

i) Legendre's differential equation $y'' - \frac{2x}{1-x^2}y' + \frac{\mu}{1-x^2}y = 0$ **(2 Marks)**

ii) Chebyshev's differential equation $(1 - x^2)y'' - xy' + n^2y = 0$. **(2 Marks)**

iii) Parametric Bessel equation $x^2y'' + xy' + (\lambda^2x^2 - m^2)y = 0$. **(2 Marks)**

b) Determine the constants $\lambda_1, \lambda_2, \lambda_3$ so that

$f(x) = \lambda_1 x + 2$

$g(x) = \lambda_2 x^2 + \lambda_3 x + 1$ and

$h(x) = x - 1$

are mutually orthogonal in $0 \le x \le 1$ and then obtain the corresponding orthogonal set. **(14 marks)**

Q4

a) Solve the higher order differential equation

$$\frac{d^3y}{dx^3} = xe^x$$ **(5 marks)**

b) Find the curves represented by the solution of $ydx + (z - y)dy + xdz = 0$, which lie in the plane $2x - y - z = 1$. **(10 marks)**

c) Verify that the differential equation below is integrable, hence solve

$zydx = zxdy + y^2dz$ **(5 marks)**

END