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MAIN EXAMINATION

SEPTEMBER - DECEMBER 2022

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS AND ACTUARIAL SCIENCE

REGULAR PROGRAMME

MAT 104: ELEMENTS OF LINEAR ALGEBRA

DATE: DECEMBER 2022	**Duration: 2 Hours**
INSTRUCTIONS: Answer Question ONE and any other TWO Questions	

Q1.

a) Given **A** and **B**, find **3A – 2b**

$$\boldsymbol{A} = \begin{pmatrix} 1 & -2 & 5 \\ 0 & -3 & 9 \\ 4 & -6 & 7 \end{pmatrix} \qquad \mathbf{B} = \begin{pmatrix} 5 & 0 & -11 \\ 3 & -5 & 1 \\ -1 & -9 & 0 \end{pmatrix}$$ **(3 Marks)**

b) Compute (**AB**)^T and **B**^{T} **A**^{T} if

$$\boldsymbol{A} = \begin{pmatrix} -2 & 1 & 0 \\ -3 & -1 & -3 \end{pmatrix} \qquad \boldsymbol{B} = \begin{pmatrix} -2 & 1 & 2 \\ -1 & -2 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$ **(4 Marks)**

c) Compute **A**^{3} if $\boldsymbol{A} = \begin{pmatrix} -2 & 3 \\ 1 & 0 \end{pmatrix}$ **(3 Marks)**

d) Find the inverse of the matrix $\boldsymbol{A} = \begin{pmatrix} 1 & 3 \\ -1 & -2 \end{pmatrix}$ if it exists. **(5 Marks)**

e) Solve the linear system with elementary row operation. **(10 Marks)**

$$-3x_1 + 2x_2 + 4x_3 = 12$$
$$x_1 - 2x_3 = -4$$
$$2x_1 - 3x_2 + 4x_3 = -3$$

f) Find the characteristic polynomial of the matrix $\boldsymbol{A} = \begin{pmatrix} -2 & 4 \\ -6 & 8 \end{pmatrix}$ **(5 Marks)**

Q2

a) Find the general solution of the homogenous system (10 Marks)

$$A = \begin{pmatrix} 1 & 2 & 2 & 1 & 4 \\ 3 & 7 & 7 & 3 & 3 \\ 2 & -5 & 5 & 2 & 9 \end{pmatrix}$$

b) Show that the vectors below are linearly independent $\boldsymbol{V_1} = \begin{pmatrix} 0 \\ 1 \\ 5 \end{pmatrix}$ and $\boldsymbol{V_2} = \begin{pmatrix} 4 \\ -1 \\ 0 \end{pmatrix}$ **(5 Marks)**

c) Compute **AB** and **BA** for the matrix given below and show if matrix multiplication is commutative. **(5 Marks)**

$$\boldsymbol{A} = \begin{pmatrix} -4 & 4 & 3 \\ 3 & -3 & -1 \\ -2 & -1 & 1 \end{pmatrix} \qquad \boldsymbol{B} = \begin{pmatrix} -1 & -1 & 0 \\ -3 & 0 & -2 \\ -2 & 1 & -2 \end{pmatrix}$$

Q3

a) Find the projection of the vector **V** = (1, 2 , 1) on the Vector **U** = (-2, 1, 3) **(4 Marks)**

b) Find the point of intersection of the plane $3x - 2y + z = -5$ and the line $x = 1 + t$, $y = -z + 2t,\ z = 4t.$ **(5 Marks)**

c) Solve the system A x = b by Cramers rule given the matrix $\boldsymbol{A} = \begin{pmatrix} 2 & 4 \\ 2 & 1 \end{pmatrix}$ and the vector $b = \begin{pmatrix} 2 \\ -4 \end{pmatrix}$ **(3 Marks)**

d) Find the parametric equation for the line of intersection of $x - y + 2z = 1$ and $x + y + z = 3$ **(3 Marks)**

e) Find the rank and nullify of the matrix $\boldsymbol{A} = \begin{pmatrix} 1 & -3 & -1 \\ -1 & 4 & 2 \\ -1 & 3 & 0 \end{pmatrix}$ **(5 Marks)**

Q4

a) Find the Reduced row echelon form of the following system of equation. **(10 Marks)**

2x + 8y + 4z = 2
2x + 5y + z = 5
4x + 10y – z = -1

b) Find the Eigen Values of the matrix $A = \begin{pmatrix} -4 & -6 & -7 \\ 3 & 5 & 3 \\ 0 & 0 & 3 \end{pmatrix}$ **(5 Marks)**

c) Given two matrices A and C below, show that C is the inverse of A **(5 Marks)**

$$A = \begin{pmatrix} 1 & -3 & 0 \\ -1 & 2 & -2 \end{pmatrix} \qquad C = \begin{pmatrix} -14 & -3 & -6 \\ -5 & -2 & -2 \end{pmatrix}$$

Q5

a) Solve if possible the matrix equation $Ax = b$ **(10 Marks)**

$$A = \begin{pmatrix} 1 & 3 & -4 \\ 1 & 5 & 2 \\ -3 & -7 & 6 \end{pmatrix} \qquad B = \begin{pmatrix} -2 \\ 4 \\ 12 \end{pmatrix}$$

b) Compute the determinant of the matrix $A_2 = \begin{pmatrix} 4 & -2 & 3 \\ 2 & 3 & 5 \\ 1 & 0 & 6 \end{pmatrix}$ **(4 Marks)**

c) Find the equation of the plane through the points (2, 4, -1) with normal vector **n** = (2, 3, 4). Find the intercept and sketch the plane. **(6 Marks)**

END