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GABA CAMPUS - ELDORET

MAIN EXAMINATION

SEPTEMBER – DECEMBER 2021 TRIMESTER

FACULTY OF SCIENCE

BACHELOR OF SCIENCE

DEPARTMENT OF MATHEMATICS AND ACTUARIAL SCIENCE

MAT 335: METHODS OF APPLIED MATHEMATICS I

Date: December 2021	**Duration:** 2 Hours
Instructions: Answer Question **ONE** and any other **TWO** Questions	

QUESTION ONE

a) The displacement of a vibrating string is described by the equation

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

With the boundary conditions

$x = 0$, $u(0,t) = 0$

$x = l$, $u(l,t) = 0$

And the initial conditions: $t = 0$, $u(x,0) = \phi(x)$

Apply the method of separation of variables to find the general solution.

(10 marks)

b) Find the general solution of the first order differential equation by use of transforms

$\frac{dx}{dt} - 2x = 4$ given that $x(0) = 1$

(6 marks)

c) Find the value of $\Gamma(\frac{5}{2})$ **(4 marks)**

d) Use Gamma function to evaluate $\int_0^\infty x^3 e^{-4x}\,dx$ **(5 marks)**

e) Use Beta function to evaluate $\int_0^1 x^5 (1-x)^4\,dx$ **(5 marks)**

QUESTION TWO

a) Prove that $L\{\cosh at\} = \frac{s}{s^2 - a^2}$ **(3 marks)**

b) Solve the boundary value problem

$$\frac{d^2x}{dt^2} - 3\frac{dy}{dx} + 2x = 2e^{3t}$$

$$x_0 = x(0) = 5$$

$$x_1 = \frac{d}{dx}\big(x(0)\big) = 7$$ **(10 marks)**

c) Prove that $L^{-1}\{t^n\} = \frac{n!}{s^{n+1}}$ **(7 marks)**

QUESTION THREE

a) Solve the boundary value problem for the Laplace equation

$$U_{xx} + U_{yy} = 0$$

Where $U(x, y)$ represents the velocity potential of fluid particle in a certain domain, particularly inside a unit circle $x^2 + y^2 > 1$. **(12 marks)**

b) Solve the differential equation
$X'' - 4X = 24Cos\ 2t$, $X_0 = 3$, $X_1 = 4$ **(8 marks)**

QUESTION FOUR

a) Apply L'Hospital Rule to calculate the Laplace transform

$$L\left\{\frac{\sin at}{t}\right\}$$ **(5 marks)**

b) Given a Beta function $\beta(m,n) = \int x^{m-1}(1-x)^{n-1}\,dx$, prove that $\beta(m,n) = \beta(n,m)$

(3 marks)

c) Find the Fourier series for the function

$$f(x)=\begin{cases}-x; & -\pi<x<0\\ 0; & 0<x<\pi\\ f(x) & =f(x+2\pi)\end{cases}$$

(12 marks)

END