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GABA CAMPUS - ELDORET

MAIN EXAMINATION

SEPTEMBER – DECEMBER 2021 TRIMESTER

FACULTY OF SCIENCE

BACHELOR OF SCIENCE

DEPARTMENT OF MATHEMATICS AND ACTUARIAL SCIENCE

MAT 204: LINEAR ALGEBRA II

Date: December 2021	**Duration:** 2 Hours
Instructions: Answer Question **ONE** and any other **TWO** Questions	

QUESTION ONE

a) Given two vectors $f(t) = t^2$ and $g(t) = t - 3t^2$. Find:

i) The inner product of the two vectors of polynomials of order ≤ 2 in an interval [0,1]. **(3 marks)**

ii) The angle between $f(t)$ and $g(t)$. **(4 marks)**

b) Determine whether the function below is a linear transformation

$T: \mathbb{R}^2 \to \mathbb{R}^2$ with $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x + y \\ y \end{bmatrix}$ **(3 marks)**

Let $T: \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation by $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ -x_2 \\ 3x_1 - 5x_2 \end{bmatrix}$

c) Find the matrix A, such that $T(\vec{x}) = A\vec{x} \quad \forall \quad \vec{x} \in \mathbb{R}^2$. **(3 marks)**

d) Suppose u and v are orthogonal vectors in V, prove that

$$\|u + v\|^2 = \|u\|^2 + \|v\|^2$$ **(3 marks)**

e) Given the system below, find the Eigen values and Eigen vectors

$2x_1 + 2x_2 = 0$

$5x_1 - x_2 = 0$ **(4 marks)**

f) Find the general solution of the system

$$\frac{dx}{dt} = x - 2y$$

And $\frac{dy}{dt} = -5x + 4y$ **(6 marks)**

g) Find the Frobenius inner product of the two matrices A and B, given that

$A = \begin{pmatrix} 3 & 9 & 14 \\ 2 & -11 & 7 \\ 3 & 5 & -6 \end{pmatrix}$ and $B = \begin{pmatrix} 19 & -5 & 5 \\ 7 & 13 & 4 \\ 2 & 12 & 8 \end{pmatrix}$ **(4 marks)**

QUESTION TWO

a) Show that the vectors $\vec{v}_1, \vec{v}_2$ and $\vec{v}_3$ are linearly independent given that

$\vec{v}_1 = (1,1,1), \vec{v}_2 = (-2,1,1)$ and $\vec{v}_3 = (0,1,-1)$. Also find a vector $\vec{A} = (5,3,1)$,

as a linear combination of $\vec{v}_1, \vec{v}_2$ and $\vec{v}_3$.

(5 marks)

b) Given the vectors $\vec{v}_1, \vec{v}_2$ and $\vec{v}_3$ as $\vec{v}_1 = (1,1,1), \vec{v}_2 = (-2,1,1)$ and $\vec{v}_3 = (0,1,-1)$, compute orthonormal basis for $\vec{v}_1, \vec{v}_2$ and $\vec{v}_3$, hence show that $A^{-1} = A^T$,

where A is a matrix whose columns are orthonormal basis of $\vec{v}_1, \vec{v}_2$ and $\vec{v}_3$.

(5 marks)

c) Apply Gram-Schmidt algorithm to orthonormalize the set of vectors

$\vec{v}_1 = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$, $\vec{v}_2 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$, $\vec{v}_3 = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$ **(5 marks)**

Examine the space V spanned by the two vectors $\vec{v}_1 = (1,2,2)$ and $\vec{v}_2 = (0,3,6)$.
Find an orthonormal basis for V.

(5 marks)

QUESTION THREE

a) Show that the vectors $\vec{v}_1, \vec{v}_2$ and $\vec{v}_3$ are linearly independent given that $\vec{v}_1 = (1,0,-1), \vec{v}_2 = (1,\sqrt{2},1)$ and $\vec{v}_3 = (1,\sqrt{2},1)$. Also find a vector $\vec{A} = (7,\sqrt{2},3)$, as a linear combination of $\vec{v}_1, \vec{v}_2$ and $\vec{v}_3$.

(4 marks)

b) Find the general solution of the system

$$\frac{dx}{dt} = y$$

And $\frac{dy}{dt} = -x$ **(7 marks)**

c) Determine the component vector of p(x) = 5+7x−3x^2 relative to the following:

i) The standard (ordered) basis B = {1, x, x^2}. **(2 marks)**

ii) The ordered basis C = {1 + x, 2 + 3x, 5 + x + x^2}. **(5 marks)**

d) Let $B = \{(3,1),\ (-2,1)\}$ and $B' = \{(2,1),\ (1,4)\}$ be ordered bases for $\mathbb{R}^2$. Obtain the transition matrix P from B-basis to B'-basis.

(4 marks)

QUESTION FOUR

a) Compute the Eigenvectors for $A = \begin{pmatrix} 7 & -2 & -4 \\ 3 & 0 & -2 \\ 6 & -2 & -3 \end{pmatrix}$ **(8 marks)**

b) Find the diagonalization of the matrix $B = \begin{pmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{pmatrix}$ **(8 marks)**

c) Given that $f(x) = 3x$ and $g(x) = 1 + x^2$. Find the angle between $f(x)$ and $g(x)$, which are vectors of polynomials of order ≤ 2 in an interval [0,1].

(4 marks)

END