# THE CATHOLIC UNIVERSITY OF EASTERN AFRICA

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**MAIN EXAMINATION**

**SEPTEMBER –DECEMBER 2021**

**FACULTY OF SCIENCE**

**DEPARTMENT OF MATHEMATICS**

**REGULAR PROGRAMME**

**MAT 700: METHODS OF APPLIED MATHEMATICS**



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|---|---|
| **Date: DECEMBER 2021** | **Duration: 3 Hours** |
| **INSTRUCTIONS: Answer ALL Questions** | |

Q1. Given the system $\begin{array}{l} \dot{x}_1 = 2x_1 \\ \dot{x}_2 = x_2 \end{array}$,

a. Find the general solution **(5 marks)**

b. Draw the phase portrait of the systems **(7 marks)**

Q2. Show that the initial value problem $\dot{x} = \frac{1}{2x}, \quad x(1) = 1$

c. has a solution $x(t)$ on the interval $(0, \infty)$ **(3 marks)**

d. that $x(t)$ is defined and continuous on $[0, \infty)$ **(4 marks)**

e. $x'(0)$ does not exist. **(3 marks)**

Q3. For the matrix $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$,

f. find the eigenvalues and eigenvectors of the matrix **(6 marks)**

g. For P, the eigenvector matrix, show that $B = P^{-1}AP$ is a diagonal matrix. **(4 marks)**

h. Solve the linear system $\dot{\vec{y}} = B\vec{y}$ and then solve $\dot{\vec{x}} = A\vec{x}$ **(10 marks)**

i. Sketch the phase portraits in both the $\vec{x}$ -plane and $\vec{y}$ -plane. **(10 marks)**

Q4. Determine if the linear system $\dot{\vec{x}} = A\vec{x}$ has a saddle, node, focus, or center at the origin and determine the stability of each node or focus (if any).

j. $\dot{\vec{x}} = \begin{bmatrix} 1 & -1 \\ 3 & 2 \end{bmatrix}\vec{x}$ **(6 marks)**

k. $\dot{\vec{x}} = \begin{bmatrix} 0 & -2 \\ 3 & 0 \end{bmatrix}\vec{x}$ **(6 marks)**

Q5. Find the first three successive approximations $u_1(t), u_2(t)$ and $u_3(t)$ for the initial value problem $\dot{x} = x^2,\ x(0) = 1$ **(6 marks)**



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