THE CATHOLIC UNIVERSITY OF EASTERN AFRICA

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MAIN EXAMINATION

SEPTEMBER –DECEMBER 2021

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS

REGULAR PROGRAMME

MAT 504: NUMERICAL ANALYSIS I

Date: DECEMBER 2021	Duration: 3 Hours
INSTRUCTIONS: Answer ANY THREE Questions All Symbols have their usual meaning.	

Q1.a) Given the Chebyshev polynomial of the first kind $T_n(x) = \cos\left(n\cos^{-1}(x)\right) = \cos(n\theta)$

where $\theta = \cos^{-1}(x)$, prove that

$$\int_{-1}^{1} \frac{T_m(x)\,T_n(x)}{\sqrt{1-x^2}}\,dx \begin{cases} 0 & ;\, m \neq n \\ \dfrac{\pi}{2} & ;\, m = n \neq 0 \\ \pi & ;\, n = m = 0 \end{cases}$$

(10marks)

b). Using Chebyshev polynomials, obtain the least square approximation of the second degree for the function $f(x) = x^4$ on the interval $[-1,1]$. **(13marks)**

Q2. a) Determine a unique Lagrange polynomial of degree 2 or less such that $f(1.0) = 14.2,\ f(2.7) = 17.8$ and $f(3.2) = 22.0$, hence using your polynomial approximate the value of $f(2.5)$. **(7marks)** b). Discuss the theory of Hermite interpolation, hence given the data below

x	$f(x)$	$f'(x)$
-1	1	-5
0	1	1
1	3	7

using the data above approximate the value of $f(0.5)$ using Hermite interpolation of degree five, $H_5(x)$. **(16marks)**

Q3.a) Given the following values of a function f of the variable t

t	0.1	0.2	0.3	0.4
$f(t)$	0.76	0.58	0.44	0.35

Obtain a least square fit of the form $f(t) = ae^{-3t} + be^{-2t}$. **(8marks)**

b). Obtain a cubic spline approximation for the function given in tabular form as

x	0	1	2	3
$f(x)$	1	2	33	244

and $m_0 = 0, \quad m_3 = 0$ on the interval $[0,1]$ hence approximate $f(0.5)$. **(9marks)**

c). Given that (x_0, f_0) and (x_1, f_1) are two adjacent tabulated points, show that the linear Lagrange interpolating polynomial of $f(x)$ is given by

$$P(x) = \frac{x - x_1}{x_0 - x_1} f_0 + \frac{x - x_0}{x_1 - x_0} f_1$$

Hence if $\sin(0.4) = 0.38942$ and $\sin(0.5) = 0.47943$ approximate the value of $\sin(0.45)$. **(6marks)**

Q4. a) Using Gram-Schmidt orthogonalization process, compute the first three polynomial $p_0(x), p_1(x), p_2(x)$ which are orthogonal on $[0,1]$ with respect to the weight function $w(x) = 1$. Hence using these polynomials obtain least square approximation of second degree for $f(x) = x^{\frac{1}{2}}$ on $[0,1]$.

(15marks)

b). Obtain a linear polynomial approximation to the function $f(x) = x^3$ on the interval $[0,1]$ using least square approximation with $w(x) = 1$. **(8marks)**

Q5.a) A square matrix of order 3 has 3 linearly independent eigenvectors, show that a matrix P can be found such that $P^{-1}AP = D$ where D is a diagonal matrix. **(6marks)**

b). Determine a matrix P which diagonalizes.

$$A = \begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$$

(13marks)

c). Use the Cayley-Hamilton theorem to find A^{-1} given $A = \begin{pmatrix} 4 & 2 \\ 1 & 1 \end{pmatrix}$. **(4marks)**

END