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MAIN EXAMINATION

SEPTEMBER –DECEMBER 2021

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS

REGULAR PROGRAMME

MAT 460: STOCHASTIC PROCESSES

Date: DECEMBER 2021	Duration: 2 Hours
INSTRUCTIONS: Answer Question ONE and any TWO Questions	

Q1. a) Obtain the probability generating function of the Poisson distribution where

$$P_k = \frac{e^{-\lambda}\lambda^k}{k!}; k = 0, 1, 2, \ldots.$$

Hence obtain the mean and variance of the distribution. **(6 marks)**

b) The arrival of customers at a store is assumed to be a Poisson process with rate parameter λ. If the store decides to close its doors after the nth customer has arrived, derive the probability density function of the length of time T that its doors remain open. **(6 marks)**

c) (i) When are two states in a Markov chain said to communicate? **(2 marks)**

(ii) Suppose that whether or not it rains today depends on previous weather conditions through the last two days. Specifically, suppose that if it has rained for the past two days, then it will rain tomorrow with probability 0.7; if it rained today but not yesterday then it will rain tomorrow with probability 0.5; if it rained yesterday but not today then it will rain tomorrow with probability 0.4; if it has not rained in the past two days then it will rain tomorrow with probability 0.2. Transform the above process into a Markov chain and determine its transition probability matrix. Classify the states of the process. State also its classes. **(6 marks)**

d) Customers arrive at a bank at a Poisson rate λ / hr. Suppose that two customers arrive during the first hour. What is the probability that;

(i) both arrived during the first 20 minutes?

(ii) at least one arrived during the first 20 minutes? **(4 marks)**

e) Consider the queuing situation with one server in which arrivals occur at the rate $\lambda = 3$ per hour and service is performed at the rate $\mu = 8$ per hour. The probabilities P_n of n customers in the system are computed for the situation as given in the table below:

n	0	1	2	3	4	5	6	7	≥ 8
P_n	0.0625	0.234	0.088	0.033	0.012	0.005	0.002	0.001	0

Obtain

(i) L_s, the expected number in the system

(ii) W_s, the expected waiting time in the system

(iii) W_q, the expected waiting time in the queue

(iv) L_q, the expected number in the queue **(6 marks)**

Q2. The difference-differential equation for a simple birth process is given by:

$$P_n'(t) = -n\lambda P_n(t) + (n-1)\lambda P_{n-1}(t), \quad n \geq 1$$

a) Suppose that the initial population is 1, that is, $P_1(0) = 1$, solve the equation for $n = 1, 2, 3$ in succession and verify that

$$P_n(0) = e^{-\lambda t}\left(1 - e^{-\lambda t}\right)^{n-1}, \quad n \geq 1$$ **(10 marks)**

b) Now if the initial population is N_0 i.e. $P_{N_0}(0) = 1$ and $P_n(0) = 0$ for $n \neq N_0$, show that

$$P_n(t) = \binom{n-1}{n-N_0} e^{-N_0\lambda t}\left(1 - e^{-\lambda t}\right)^{n-N_0}, \quad n \geq N_0$$ **(10 marks)**

Q3. a) Consider the queuing model $(M/M/1):(GD/\infty/\infty)$

(i) Explain clearly the meaning of each symbol in the model **(3 marks)**

(ii) If in such a model $P_n = (1-\rho)\rho^n$ where

$\rho = \lambda/\mu < 1$, $n = 0, 1, 2, \ldots$

Show that $L_s = \dfrac{\rho}{1-\rho}$ where L_s is the expected waiting time in the system **(7 marks)**

b)A fast food restaurant has one drive in window. It is estimated that cars arrive according to a Poisson distribution at the rate of 2 every 5 minutes and that there is enough space to accommodate a line of 10 cars. Other arriving cars can wait outside this space if necessary. It takes 1.5 minutes on the average to fill an order but the service time actually varies according to an experimental distribution. Determine:

(i) The probability that the facility is idle

(ii) The expected number of customers waiting but currently not being served

(iii)The expected waiting time until a customer can place his order at the window

(iv)The probability that the waiting time will exceed the capacity of the space leading to the drive-in window **(10 marks)**

Q4. a) A food facility packs its products in 1 kg packages. The packages are subjected to a quality inspection procedure to detect defective ones. If it is known that the proportion of the defective packages is 0.02, determine:

(i) the probability that the number of good inspections necessary until the first defective package is found is at least two.
(ii) The probability that the number of good packages inspected before detecting the first defective package is at least two.
(iii) The expected number of inspections until the first defective package is found.
(iv) The expected number of good packages inspected before the first defective package is found.

(10marks)

b) The occurrence of misprints in a book of 500 pages is believed to be a Poisson process with rate of 0.8 misprints/ page.

Compute:

(i) The probability of no misprints in 5 randomly selected pages of the book

(ii) The average number of pages separating any two misprints

(iii) The probability of 8 misprints in less than ten pages (just give the expression without exact computation)

(iv) The average number of pages that must be examined in order to fund 80 misprints. **(10 marks)**

Q5. a) Obtain the steady state probabilities for the following transition matrix

$$P = \begin{bmatrix} 0.7 & 0.2 & 0.1 & 0 \\ 0.3 & 0.4 & 0.2 & 0.1 \\ 0 & 0.3 & 0.4 & 0.3 \\ 0 & 0 & 0.3 & 0.7 \end{bmatrix}$$

(10 marks)

b) Let $P = \begin{bmatrix} 0 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$ be a matrix of transition probabilities for a two state Markov chain.

Find P^n and hence obtain $\lim_{n \to \infty} P^n$. **(10 marks)**

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