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MAIN EXAMINATION

SEPTEMBER –DECEMBER 2021

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS

REGULAR PROGRAMME

MAT 462: OPERATIONS RESEARCH II

Date: DECEMBER 2021	**Duration: 2 Hours**
INSTRUCTIONS: Answer Question ONE and any TWO Questions	

Q1.

a) Define the following terms as used in Operations Research

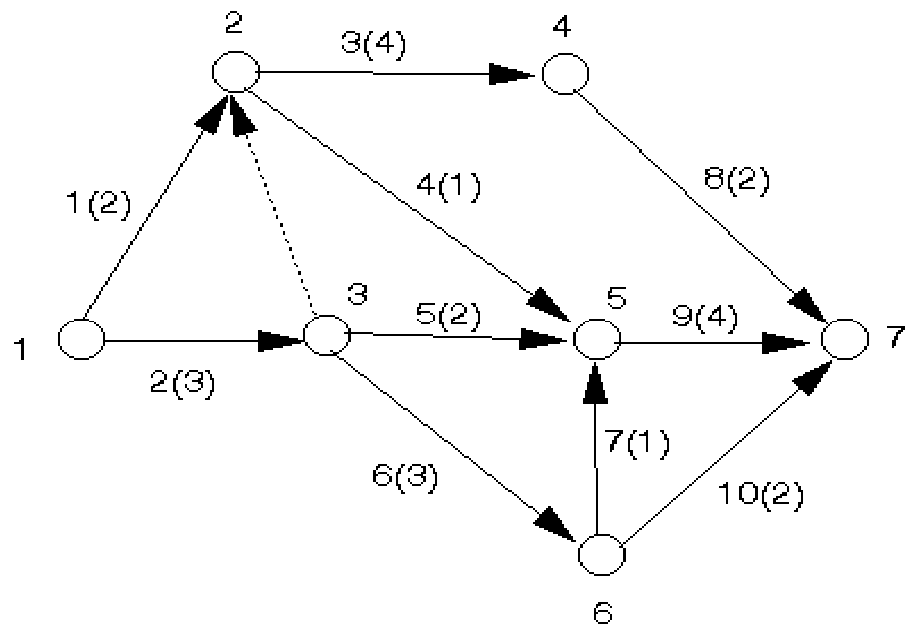
i. Feasible solution **(1mark)**
ii. Non-degenerate basic feasible solution **(1mark)**
iii. Queuing system. **(2marks)**
iv. Network **(2marks)**

b) Give the mathematical formulation of an Assignment Problem **(3marks)**

c) Distinguish between Transport Problem and Assignment Problem **(2marks)**

d) Distinguish the terms Earliest Start Time and Latest Start Time **(3marks)**

e) For the activity on arc network shown below calculate the earliest and latest start times and hence find the float for each activity, the critical activities and the overall project completion time. **(6marks)**



How would the overall project completion time be affected by an increase in the completion time for activity 1 of 2 weeks? **(2marks)**

f) Customers arrive at a bank at a Poisson rate λ / hr. Suppose that two customers arrive during the first hour. What is the probability that;

i. Both arrived during the first 20 minutes? **(2marks)**

ii. At least one arrived during the first 20 minutes? **(2marks)**

g) Construct a network for the project whose activities and their precedence relationships are given below;

Activity	A	B	C	D	E	F	G	H	I
Immediate Predecessor	-	-	A,B	B	B	A,B	F,D	F,D	C.G

(4marks)

Q2.

a) The following table shows the jobs of a network along with their time estimates. The time estimates are in days:

Job	**1-2**	**1-6**	**2-3**	**2-4**	**3-5**	**4-5**	**5-8**	**6-7**	**7-8**
a	3	2	6	2	5	3	1	3	4
m	6	5	12	5	11	6	4	9	19
b	15	14	30	8	17	15	7	27	28

i. Draw the project network

(6marks)

ii. Find the critical path

(3marks)

iii. Find the probability that the project is completed in 31 days.

(4marks)

b) A Television repairman finds that the time spent on his jobs has an exponential distribution with mean 30 minutes. If he repairs sets in the order in which they come in and the arrival is approximately Poisson with an average rate of 10 per 8-hour day, what is the repairman's expected idle time each day?

(7marks)

Q3.

a) Find the initial basic feasible solution for the following transportation problem using the Vogel's Approximation Method

Destination						
Factory		D_1	D_2	D_3	D_4	**Supply**
	F_1	3	3	4	1	100
	F_2	4	2	4	2	125
	F_3	1	5	3	2	75
	Demand	120	80	75	25	300

(10 marks)

b) Arrivals in a telephone booth are considered to be Poisson with an average time of 10 minutes between one arrival and the next. The length of a telephone call is assumed to be distributed exponentially with mean 3 minutes.

i. What is the probability that a person arriving at the booth will have to wait? **(3marks)**

ii. What is the average length of the queue that forms from time to time? **(3marks)**

iii. The telephone department will install a second booth when convinced that an arrival would except to have to wait at least three minutes for the phone. By how much must the flow of arrivals be increased in order to justify second booth? **(4marks)**

Q4.

a) Four different jobs can be done on four different machines and take down time costs are prohibitively high for change over's. The matrix below gives the cost in Kenya Shillings ('000) of producing job i on machine j:

Jobs	Machine			
	M_1	M_2	M_3	M_4
J_1	5	7	11	6
J_2	8	5	9	6
J_3	4	7	10	7
J_4	10	4	8	3

How should the jobs be assigned to the various machines so that the total cost is minimized.

(10marks)

b) Consider the queuing model $(M/M/1):(\infty/FCFS)$

i. Explain clearly the meaning of each symbol in the model **(3marks)**

ii. If in such a model $p_n = (1-\rho)\rho^n$ where $\rho = \lambda/\mu < 1, n = 1, 2, \ldots$, show that $L_s = \frac{\rho}{1-\rho}$ where L_s is the expected number of units in the system. **(7marks)**

Q5.

a) Tasks A,B,...,H ,I constitute a project. The notation X<Y means that the task X must be completed before Yis started.
With the notation,

$$A < D, A < E, B < F, D < F, C < G, C < H, F < I, G < I$$

Draw a graph to represent the sequence of tasks and find the minimum time of completion of the project, when the time (in days) of completion of each task is as follows.
The above constraints can be given in the following table

Task	A	B	C	D	E	F	G	H	I
Time (days)	8	10	8	10	16	17	18	14	9

(10marks)

b) Use dynamic programming to solve the Linear Programming Problem

$Max.Z = x_1 + 9x_2$

Subject to the constraints

$2x_1 + x_2 \le 25$

$x_2 \le 11$

$x_1, x_2 \ge 0$

(10marks)

END