THE CATHOLIC UNIVERSITY OF EASTERN AFRICA
A. M. E. C. E. A

MAIN EXAMINATION

SEPTEMBER -DECEMBER 2021

FACULTY OF SCIENCE
DEPARTMENT OF MATHEMATICS
REGULAR PROGRAMME
Date: DECEMBER 2021 Duration: 2 Hours

INSTRUCTIONS: Answer Question ONE and any TWO Questions

Q1.
a) A statistical quality controller uses the $\bar{X}$-chart for monitoring a quality characteristic of a product. If the process mean $(\mu)$ and process standard deviation ( $\sigma$ ) are 100 and 5 , respectively, find the centre line and control limits for the $\bar{X}$-chart. It is given that $A=1.5$ for $n=4$.
b) In a company manufacturing cricket ball, the quality controller inspects the balls and classifies them as defective or non-defective on the basis of certain defects. The company manager wants to maintain the process so that an average of not more than 5 percent of the output is not defective.
i) Suggest a suitable control chart for this purpose.
(1 Mark)
ii) If the company can work with a sample size 500, calculate the center line and control limits for this chart.
(5 Marks)
c) Suppose a cricket ball manufacturing company supplies lots of 500 balls. To check the quality of the lots, a buyer draws a random sample of size 20 balls from each lot and accepts the lot if the inspected sample contains at most one defective ball. Otherwise he/she rejects the lot. If the lot consists of 10 defective balls, find:
i) The lot qualiy $p$.
ii) The probability of accepting the lot, $p_{a}$.
iii) Suppose the manufacturing company and the buyer agree that $A Q L=0.02$. find the producer's risk for this plan.
iv) Suppose the manufacturing company and the buyer agree that $L T P D=0.05$. Find the consumers risk for this plan.
(2 Marks)
d) A manufacturer of silicon chips produces lots of 1000 chips for shipment. A buyer uses a double sampling plan with $n_{1}=5, c_{1}=0, n_{2}=20, c_{2}=2$ to test the quality of the lots. Explain the procedure for implementing it under acceptance sampling plan.

## Marks)

Q2.
a) A manufacturer of silicon chips produces lots of 1000 chips for shipment. A buyer uses a double sampling plan with $n_{1}=5, c_{1}=0, n_{2}=20, c_{2}=2$ to test the quality of the lots. Explain the procedure for implementing it under acceptance sampling plan. Suppose the incoming quality of the lot is 0.03 . Calculate the probabilities of accepting the lot on the first sample and on the second sample. What is the probability of accepting the lot?
(20 Marks)

Q3.
b) The following data are found during the inspection of the first 15 samples of size 100 each from a lot of two-wheelers manufactured by an automotive company.

| Sample No. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| No. of defectives | 3 | 4 | 6 | 2 | 12 | 5 | 3 | 6 | 3 | 5 | 4 | 15 | 5 | 2 | 3 |

Draw the chart for fraction defective $(p)$ and comment on the state of control.

## Marks)

a) A hospital receives disposable injection syringes in lots of 2000. A single sampling plan with $n=25$ and $c=2$ is being used for inspection by the quality inspector of the hospital. Suppose the acceptance quality ( $A Q L$ ) and lot tolerance percent defective ( LTPD) are 0.04 and 0.10 respectively. Calculate the producer's risks and consumer's risk for this plan.
(7 Marks)

Q4.
A shopkeeper purchases pens from a pen company in cartons (lots) that usually contain one thousand pens. To check the quality of the pens, the shopkeeper
selects 25 pens at random from each carton and visually inspects each selected pen for certain defects. The shopkeeper accepts the lot if the inspected sample contains at most two defective pens, otherwise he/she rejects the lot. If there are 40 defective pens in each carton:
i) Find $p$.
(1 Mark)
ii) Find $p_{a}$.
iii) If the shopkeeper and the company have decided that $A Q L=0.03$ and $L T P D=0.10$, calculate the producers risk and the consumers risk.

## (7 Marks)

iv) For sampling plan with $n=5$ and $c=0$, find the probability of accepting a lot that has $2 \%$ defective units by assuming that the number of defective units in a sample follows a binomial distribution.
(2 Marks)
v) If the acceptance quality level ( $A Q L$ ) and the lot acceptance percent defective ( LTPD) are $1 \%$ and $5 \%$ respectively, calculate the producer's risk and the consumer's risk for this plan.
(7 Marks)

Q5.
Suppose a mobile phone company produces mobile phones in lots of 400 phones each. To check the quality of the lots, the quality inspector of the company uses $s$ double sampling plan with $n_{1}=15, c_{1}=1, n_{2}=30, c_{2}=3$. Suppose the incoming quality of the lot is 0.05 .
a) What is the probability of accepting the lot on the first sample?
(2 Marks)
b) What is the probability of final acceptance?
c) Suppose the rejected lots are screened and all defective mobiles are replaced by non-defective ones.
d) Calculate the average outgoing quality (ASN) for this plan.

## *END*

