# THE CATHOLIC UNIVERSITY OF EASTERN AFRICA

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**MAIN EXAMINATION**

**SEPTEMBER –DECEMBER 2021**

**FACULTY OF SCIENCE**

**DEPARTMENT OF MATHEMATICS**

**REGULAR PROGRAMME**

**MAT 307: REAL ANALYSIS III**

| | |
|---|---|
| **Date: DECEMBER 2021** | **Duration: 2 Hours** |
| **INSTRUCTIONS: Answer Question ONE and any TWO Questions** | |

Q1.

a. Show thatif $f$ is increasing on $[a,b]$, then $f$ is of bounded variation on $[a,b]$ and $V(f,[a,b])=f(b)-f(a)$. **[5marks]**

b. Prove that if $F$ is differentiable with $F'=f$ continuous; then if $g$ is integrable

$$\int_I g(u)\,dF(u)=\int_I g(u)f(u)\,du$$ **[5marks]**

c. Prove that a sequence $(f_n)$ of bounded functions on $A\subset R$ converges uniformly on A to $f$ *iff* $\vee|f_n-f|\vee\to 0$. **[5marks]**

d. State the root test outlining clearly its condition for absolute convergence and divergence **[5marks]**

e. Test the convergence of the following

i. $\displaystyle\sum_0^\infty \frac{2^{2n}3^n}{10^n}$ **[5marks]**

ii. $\displaystyle\sum_{n=2}^\infty \frac{1}{n\ln n}$ **[5marks]**

Q2.

a. State the fundamental theorem of calculus **(First Form)** and prove it **[9marks]**

b. Evaluate the following by fundamental theorem

i. $G(x) = \arctan x \; \forall x \in [a, b]$ **[3marks]**

ii. $A(x) = |x| \; \forall x \in [-10, 10]$ **[4marks]**

c. Show that $\left(\frac{1}{n}\right)$ is a Cauchy sequence **[4marks]**

Q3.

a. Suppose γ is a circle centered about the origin, oriented counter-clockwise. Then,

$\int_{\gamma} z^{-1} dz = 2\pi i$. **[10marks]**

b. Show that the function $f$ defined by

$f(x) = ¿$

is not of bounded variation **[10marks]**

Q4.

a. Determine the radius and interval of convergence of the series **[5marks]**

$$\sum_{n=0}^{\infty} \frac{(x-4)^n}{5^n}$$

b. Use the fourth Maclaurin polynomial to approximate $\ln(1.1)$ **[7marks]**

c. Show that if $f \in R[a, b]$ then the value of the integral is uniquely determined **[8marks]**



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