THE CATHOLIC UNIVERSITY OF EASTERN AFRICA

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MAIN EXAMINATION

SEPTEMBER –DECEMBER 2021

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS

REGULAR PROGRAMME

MAT 308: NUMBER THEORY

Date: DECEMBER 2021	**Duration: 2 Hours**
INSTRUCTIONS: Answer Question ONE and any TWO Questions	

Q1.

a. Define a prime number.
(2 marks)

b. Find the $lcm(272,1479)$.
(3 marks)

c. Find the integers (x,y) such that $gcd(a,b)=ax+by$ for the pair $(398,270)$.
(3 marks)

d. Find three different Pythagoras triples of the form $(16,y,z)$.
(3 marks)

e. Find all solutions in positive integer of the Diophantine equation $x^2+2y^2=z^2$
(4 marks)

f. Find all primitive Pythagoras triples (a,b,c) with $z\leq 40$.
(3 marks)

g. Use the Euclidean algorithm to obtain integers x and y satisfying $gcd(56,72)=56x+7y$.
(3 marks)

h. Express $gcd(6,10,15)$ as a linear combination of its element integers.
(3 marks)

i. Find $gcd\ (0,0,1001)$.
(3 marks)

j. Find three naturally relative prime integers from among the integers $(66,105,42,70,\wedge 165)$.

(3 marks)

Q2.

a. Find all the primes less than 100 using the sieve of Eratosthenes.

(5 marks)

b. Using Fermat's little theorem, find the least positive residue of $2^{1000000}$ modulo 17.

(5 marks)

c. Find the prime factorization of the integers $(1234,10140,36000)$.

(5 marks)

d. Show that if k is a positive integer, then $3k+2$ and $5k+3$ are relatively prime integers greater than 1.

(5 marks)

Q3.

a. If $k>0$ then $gcd(ka,kb)=k\,gcd\ (a,b)$. Prove.

(5 marks)

b. For positive integers a and b, $gcd(a,b)lcm(a,b)=ab$. Prove.

(5 marks)

c. If $gcd(a,b)=d$, then $gcd\left(\frac{a}{d}.\frac{b}{d}\right)=1$. Prove that $\frac{a}{d}$ and $\frac{b}{d}$ are relatively prime.

(5 marks)

d. If $a \vee b$and $b \neq 0$, then ¿$a \vee \leq \vee b \vee$¿.

(5 marks)

Q4.

a. Let $m=5$ and $n=2$, so that $(m,n)=1$, $m \neq n (mod\, 2)$ and $m>n$. Find the primitive Pythagorean triple.

(5 marks)

b. Generate a table of primitive Pythagorean triples with $m \leq 6$.

(5 marks)

c. If (a,b,c) is a primitive Pythagorean triple. Illustrate that a and b are of opposite parity.

(10 marks)

Q5.

a. Using Wilson's theorem, find the least positive residue of $8 \times 9 \times 10 \times 11 \times 12 \times 13$ modulo 7.

(5 marks)

b. Periodical cicadas are insects with very long larva periods and brief adult lives. For each species of periodical cicadas with larva period of 17 years, there is a similar species with a larva period of 13 years. If both, the 17-year and 13-year old species emerged in a particular location in 1900, when will they both emerge in that location?

(5 marks)

c. If $a \vee b$ and $a \vee c$, then $a \vee (bx + cy)$ for arbitrary integers x and y.

(5 marks)

d. Show that if (a, b, c) is a Pythagorean triple, then a or b is divisible by 3.

(5 marks)

END