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MAIN EXAMINATION

SEPTEMBER –DECEMBER 2021

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS

REGULAR PROGRAMME

MAT 333: DIFFERENTIAL GEOMETRY

Date: DECEMBER 2021	**Duration: 2 Hours**
INSTRUCTIONS: Answer Question ONE and any TWO Questions	

Q1. a) If $\vec{A} = xz^3 i - 2x^2 yzj + 2yz^4 k$,find $\nabla \times \vec{A}$ (or curl $\vec{A}$¿ at point (1,-1,1). **(3 marks)**

b) A particle moves along a curve whose parametric equations are $x = e^{-t}$, $y = 2\cos 3t$, $z = 3\sin 3t$, where t is the time.

i) Determine its velocity and acceleration at any time t. **(3 marks)**

ii) Find the magnitudes of the velocity and acceleration at t=0.

(3 marks)

c. Prove that

i)$\nabla^2 \left(\frac{1}{r} \right) = 0$ **(4 marks)**

ii) Curl grad $\phi = 0$ **(3 marks)**

iii) ¿$curl\,\vec{A} = 0$ **(3 marks)**

Where symbols have their usual meaning.

d. Find the total work done in moving a particle in a force field given by $\vec{F} = 3xyi - 5zj + 10xk$along the curve $x = t^2 + 1, y = 2t^2, z = t^3$ from t=1 to t=2. (5 marks)

e. i) If $\vec{A} = x^2 zi - 2y^3 z^2 j + xy^2 zk$, find (¿$\vec{A}$) at the point (1,-1,1). **(3 marks)**

ii) Show that $\nabla.\nabla\phi=\nabla^2\phi$, where $\phi=2x^3y^2z^4$. **(3 marks)**

Q2. i) Find $\nabla\phi$ if $\phi=\log|\vec{r}|$ **(5 marks)**

ii) If $\vec{v}=\vec{w}\times\vec{r}$ prove $\vec{w}=\frac{1}{2}curl\,\vec{v}$ where $\vec{w}$ is a constant vector. **(4 marks)**

iii) Let $\vec{F}=2xzi-xj+y^2k$, evaluate $\iiint\vec{F}\,dv$ where v is the region bounded by surfaces

$x=0, x=2, y=0, y=6, z=x^2, z=4.$ **(10 marks)**

Q3. a) i) Find a unit tangent vector to any point on the curve $x=t^2+1, y=4t-3, z=2t^2-6t$

(6 marks)

ii) Determine the unit tangent vector at the point t=2. **(2 marks)**

b) Evaluate $\int\vec{A}\times\frac{d^2\vec{A}}{dt^2}dt$. **(2 marks)**

c) If $\vec{A}=(3x^2+6y)i-14yzj+20xz^2k$, evaluate $\int_c\vec{A}.d\vec{r}$ from (0,0,0) to (1,1,1) along the following path $x=t, y=t^2$, z¿t^3 **(10 marks)**

Q4. a) Given the space curve $x=t, y=t^2, z=\frac{2}{3}t^3$. Find

i) The curvature k

ii) Radius of the curvature **(10 marks)**

b) Express the divergence theorem in words and write it in rectangular form. **(5 marks)**

c) Find a unit normal to the surface $x^2y+2xy=4$ at the point (2,-2,3). **(5 marks)**

Q5. a) Prove that

i) $\frac{d\hat{T}}{ds}=k\hat{N}$

ii) $\frac{d\hat{B}}{ds}=-\tau\hat{N}$

iii) $\frac{d\hat{N}}{ds}=-\tau\hat{B}-k\hat{T}$ **(10 marks)**

b) i) Show that $\vec{F}=(2xy+z^3)i+x^2j+3xz^2k$ is a conservative force field.

ii) Find a scalar potential

iii) Find the work done in moving an object in this field from (1, -2, 1) to (3, 1,

(10 marks)

END