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MAIN EXAMINATION

SEPTEMBER –DECEMBER 2021

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS

REGULAR PROGRAMME

MAT 332: NUMERICAL ANALYSIS I

Date: DECEMBER 2021	**Duration: 2 Hours**
INSTRUCTIONS: Answer Question ONE and any TWO Questions	

Q1. a) Describe the following types of errors:

i) Initial error. **(1mark)**

ii) Local truncation error. **(1mark)**

iii) Local round-off error. **(1mark)**

b) Use Newton-Raphson method to find the only real root of the equation $x^3 - x - 1$ correct to 9 decimal places. **(3marks)**

c) Let $f(x) = \ln(x+5) - \ln(5)$. Approximate this function by $\frac{x}{5} - \frac{x^2}{50}$. Compute the error in this estimate provided $|x| < 0.1$ using Taylor series approach in the vicinity of $x = 0$.

Hint: the bound of error is given by $|R_n| = \left| \frac{f^{n+1}(c)}{(n+1)!}(x-a)^n \right|$ when $a < c < x$. **(3marks)**

d) Prepare a table of forward differences for the function $f(x) = x^3 + 5x - 7$ for $x = -1, 0, 2, 3, 4, 5.$ Continue the table to extrapolate $f(6)$. **(4marks)**

e) The function $f(x) = e^x - 3x^2$ has three roots. An obvious arrangement is $x = \pm\sqrt{\frac{e^x}{3}}.$ Show beginning with $x_0 = 0,$ that this arrangement will converge to a root near $x = -0.5$ if the negative value is used, and that it converges to a root near 1.0 if the positive value is used. Show however, that this form does not converge to the third root near 4.0 even when nearly exact starting value is used. Find alternative form which will converge to the root near 4.0. **(6marks)**

f) Prove the following:

i) $1 + \mu^2\delta^2 = \left(1 + \frac{1}{2}\delta^2\right)^2$ **(2marks)**

ii) $\left(\Delta - \frac{1}{2}\delta^2\right) = \delta\left(1 + \frac{\delta^2}{4}\right)^{\frac{1}{2}}$ **(3marks)**

g) Use Trapezoidal rule to calculate $I = \int_0^1 \frac{dx}{1+x}$ correct to 3 decimal place, taking $h = 0.25$. **(3marks)**

h) Use Newton's advancing difference formula to find a cubic polynomial which takes the following values.

x	0	1	2	3
$f(x)$	1	0	1	10

Hence or otherwise find $f(4)$. **(4marks)**

Q2. a) Define the operators Δ and ∇, hence show that $\Delta^r y_k = \nabla^r y_{k+r}$. **(4marks)**

b) Use a suitable re-arrangement of the function $f(x) = x^2 - 2x - 3 = 0$ to compute the first three iterates of the root near $x_0 = 4.$ Apply Aitken's acceleration scheme to compute root of the function near $x_0 = 4$, making use of the available three values of x. **(4marks)**

c) Show that $f(x) = x^3 + 4x^2 - 10 = 0$ has a root in the interval $[1,2]$ and use bisection method to determine an approximation to the root that is accurate to at least within 10^{-4}. Explain your working. Compute the iterates to 6 decimal place. **(12marks)**

Q3. a) Use Newton-Raphson method to find the root of the equation

$f(x) = x^2 - 2xe^{-x} + e^{-2x} = 0$

Take the starting guess $x_0 = 0.5$ and give your answer to 4 decimal place.

(5marks)

b) The function $y = f(x)$ is given by the points (7,3), (8,1), (9,1) and (10,9). Find the value of y for $x = 9.5$ using cubic Lagrange's interpolation Formula. The data in tabulated form is:

x	7	8	9	10
$f(x)$	3	1	1	9

(6marks)

c) A function $f(x) = y$ is given by the table below,

x	2.94	2.96	2.98	3.0	3.02	3.04	3.06
$f(x)$	0.1826	0.1811	0.1797	0.1783	0.1769	0.1755	0.1742

Find the second derivative at $x=3$ or $f''(3)$. **(9marks)**

Q4. a) Derive Newton's Gregory Forward Interpolation Formula (NGFIF). **(6marks)**

b) Given that $\sin(45^\circ)=0.7071, \sin(50^\circ)=0.7660, \sin(55^\circ)=0.8192, \sin(60^\circ)=0.8660$,

find $\sin(52^\circ)$ using Newton's Gregory Forward Interpolation Formula.**(5marks)**

c) Find the value of y when $x=372.1$ from the following data:

x	361	367	378	387	399
$y(x)$	154.9	167	191	212.5	244.2

Use Lagrange's interpolation Formula for unequal interval taking $x_0=361$, $x_1=367, x_2=378, x_3=387$ and $x_4=399$. **(9marks)**

Q5. a) Given $U_0=1, U_1=11, U_2=21, U_3=28$ and $U_4=29$ find $\Delta^4 U_0$ without constructing the finite difference table. **(4marks)**

b) Use Trapezoidal Rule to evaluate the appropriate value of the following definite integral.

$$\int_1^7 f(x)dx$$

Given that,

x	1	2	3	4	5	6	7

$f(x)$	2.105	2.808	3.614	4.604	5.857	7.451	9.467

(4marks)

c) Apply Simpson's $1/3$ rule to evaluate the integral $I = \int_0^6 \frac{dx}{1+x^2}$ to 6 decimal places by dividing the range into 6 equal parts. Give your answer to 5 decimal place. **(5marks)**

d) Apply Simpson's $3/8$ rule to work out $\int_{0.2}^{1.4} (\sin(x) - \ln(x) + e^x) dx$ using 12 subdivisions of $h = 0.1$. **(7marks)**

END