THE CATHOLIC UNIVERSITY OF EASTERN AFRICA

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MAIN EXAMINATION

SEPTEMBER –DECEMBER 2021

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS

REGULAR PROGRAMME

MAT 101: DIFFERENTIAL CALCULUS

Date: DECEMBER 2021	**Duration: 2 Hours**
INSTRUCTIONS: Answer Question ONE and any TWO Questions	

Q1. a). Find the slope of the tangent to $x^2 + y^3 = 5$ at the point when $x = 2$. **(4marks)**

b). Air is being pumped into a spherical balloon at a rate of $5cm^3 / \min$.
Determine the rate at which the radius of the balloon is increasing when the diameter of the balloon is $20cm$. **(4marks)**

c). Find $\frac{dy}{dx}$, given $y = \frac{u^2 - 1}{u^2 + 1}$ and $u = \sqrt[3]{x^2 + 2}$. **(4marks)**

d). Find from first principle, the derivative of the function $y = \sqrt[3]{x}$. **(4marks)**

e). A point moves along the curve $y = x^3 - 3x + 5$ so that $x = \frac{1}{2}\sqrt{t} + 3$, where t is time. At what rate is y changing when $t = 4$. **(3marks)**

f). Use Newton's-Raphson method to find the root of the given equation, accurate to six decimal places, that lies in the given interval
$2x^2 + 5 = e^x$ in $[3,4]$. **(3marks)**

g). (i) State Mean Value Theorem. **(2marks)**

(ii) If $f(x)=4x^3-8x^2+7x-2$, determine whether or not $f(x)$ satisfies the hypotheses of the Mean Value Theorem on $[2,5]$ and if it does, find the values in $[2,5]$ which satisfy the conclusions of the theorem. **(4marks)**

h). Show that the ellipse $x^2+2y^2=2$ and the hyperbola $2x^2-2y^2=1$ intersect at right angles. **(2marks)**

Q2. a) (i) Find from first principles, the derivative of $y=\tan(2x)$. **(7marks)**

(ii) Prove that the lines tangent to the curves $5y-2x+y^3-x^2y=0$ and $2y+5x+x^4-x^3y^2=0$ at the origin intersect at right angles. **(6marks)**

b) The production costs, in dollars, per week of producing x widgets is given by,

$C(x)=4000-32x+0.08x^2+0.00006x^3$

and the demand function for the widgets is given by,

$p(x)=250+0.02x-0.001x^2$

Find:

(i) Marginal Cost. **(3marks)**
(ii) Marginal revenue. **(2marks)**
(iii) Marginal profit. **(2marks)**

when $x=200$ and $x=400$? what do these numbers tell you about the cost, revenue and profit?.

Q3. (a) Find and classify the stationary points of $y=xe^{-x^2}$. Sketch the graph. **(5marks)**

(b) Water is pouring into a leaky tank at the rate of $10m^3$ per hour. The tank is a cone with vortex down $9m$ in depth and $3m$ in radius at the top. Show that when the depth is h meters the volume of water in the tank is $V=\frac{1}{27}\pi h^3$. The surface of the water in the tank is rising at the rate of $20cm$ per hour when the depth is $6m$. How fast is the water leaking out at that time? **(6marks)**

(c) Using implicit differentiation, derive an expression for $\frac{dy}{dx}$ where x and y are defined by the solution to the expression $y^2-x^3=2xe^y$. **(3marks)**

(d) Find the Taylor series expansion for $y=\cos(x)$ about $x=0$ for terms up to $O(x^7)$ then write down the general formula for its series expansion. **(6marks)**

Q4.a) Find the derivatives of:

(i) $y=4x^5\sin(x)e^x$. **(3marks)**

(ii) $\frac{x-y}{x+y}=\frac{x^2}{y}+1, \quad x\neq y, \quad y\neq 0.$ **(4marks)**

(b) A management company is planning to build a new apartment complex. They know that if the complex contains x apartments the maintenance costs for the building, landscaping etc. will be,

$C(x)=4000+14x-0.04x^2,$

the land they have purchased can hold a complex of at most 500 apartments. How many apartments should the complex have in order to minimize the maintenance costs? **(5marks)**

(c) Find from first principle, the derivative of $y=\sqrt{x}+\frac{1}{\sqrt{x}}$. **(8marks)**

Q5. (a) For the equation of a circle of radius a and center $(0,0)$,

that is, $x^2+y^2=a^2$. Show that,

$y'' = -\frac{a^2}{y^3}$

(4marks)

(b). i). Find the Taylor polynomials of up to $O(x^5)$ to approximate

$f(x) = \frac{1}{1-x}$ near $x = 0$. **(5marks)**

(ii). In particular, use the expansion found in **b(i)** above to find the accuracy of the Taylor polynomial approximations of

$f(x) = \frac{1}{1-x}$, at $x = 0.1$ to 5 decimal places. **(3marks)**

(c). Find the derivatives of y with respect to x if:

(i). $\cos(x^2 + 2y) + xe^{y^2} = 1$. **(4marks)**

(ii). $\tan(x^2 y^4) = 3x + y^2$. **(4marks)**

END