# THE CATHOLIC UNIVERSITY OF EASTERN AFRICA

## A. M. E. C. E. A

P.O. Box 62157

00200 Nairobi - KENYA

Telephone: 891601-6

Ext 1022/23/25


**MAIN EXAMINATION**

**SEPTEMBER –DECEMBER 2021**

**FACULTY OF SCIENCE**

**DEPARTMENT OF MATHEMATICS**

**REGULAR PROGRAMME**

**ACS 301: ACTUARIAL MATHEMATICS II**

| Date: DECEMBER 2021 | Duration: 2 Hours |
|---|---|
| INSTRUCTIONS: Answer Question ONE and any TWO Questions | |



Q1.

a) By definition, differentiate the random variables, $T_{xy} \wedge T_{\overline{xy}}$ **4marks**

b) Explain the mortality of AM92 (ultimate) for both lives, calculate the following

i. $q_{66:65}$ **4marks**

ii. $\mu_{38:30}$ **3marks**

c) Simplify the sum ${}^{\square}_{n}q^{1}_{xy} + {}^{\square}_{n}q^{1}_{xy}$, and use the result to prove ${}^{\square}_{n}q^{1}_{xx} = \frac{1}{2}\,{}^{\square}_{n}q^{\square}_{xx}$ **4marks**

d) A joint life assurance contract provides death benefit of Kshs. 100,000 payable immediately on the second death of the two lives, a male life currently aged 60 and a female life currently aged 55 exact.

Calculate the expected present value of this contract.

Basis: Mortality PMA92C20 (male life), PFA92C20 (female life)

Interest: 4% per annum

Expenses: Nil **5marks**

e) Calculate $5 \vee 3^{q^{1}_{40:40}}$

Basis: Mortality AM92 **5marks**

f) Why would an insurance firm keep reserves for joint-life status policies **5 marks**

Q2.

a) In a mortality table known to follow Makeham's law you are given that A=0.03 , $c^{10}$ = 5

and $e^{0}_{30:60}=17$ Calculate ${}_{x}q^{\;1}_{30:60}$ **10marks**

b) Given that $\mu_x=\frac{1}{100-x}$ For $0\leq x\leq 100$, calculate the value of ${}_{30}q^{\;2}_{50:60}$ **10marks**

Q3.

a) Marge and Homer, both aged 55 exact, take out a policy that provides a lump sum of £50,000 payable immediately on the second death. Premiums are payable annually in advance while at least one of Marge and Homer is alive. Calculate the annual premium for the policy assuming PA92C20 mortality, 4% pa interest, and no expenses. **8marks**

b) The random variable $T_{xy}$ Represents the time to failure of the joint life status (xy). (x) is subject to a constant force of mortality of 0.01 and (y) is subject to a constant force of mortality of 0.02. Calculate the value of $E(T$ ¿¿ $xy)$¿ assuming that (x) and (y) are independent with respect to mortality. **8 marks**

c) Define the following terms:

i. Contingent assurances **2marks**

ii. Reversionary annuities **2marks**

Q4.

a)

i. Express fully in words:

**4marks**

ii. Express $\bar{a}_{\overline{xy}:\overline{n}|}$ as the expected value of random variables and hence show that

**4marks**

b) Jim and Dot, both aged 60, buy an annuity payable monthly in advance for at most 20 years, which is payable while at least one of them is alive. Calculate the expected present value of the annuity assuming 4% pa interest and PA92C20 mortality. **12marks**

Q5.

a) The following assurance functions represent the factors used for valuing the benefits from annual premium insurance contracts:

i. $A^{1}_{xy}$

ii. $A^{2}_{xy}$

Specify the appropriate annuity factor to be used in valuing premiums **6marks**

b) A life insurance company issues a special endowment assurance policy for a 25 year term to two lives (x) and (y) under the policy, sum assured of 100,000 is paid immediately on the death of the second life within the 25 year term. At the end of the 25years a sum assured of 50,000 is paid to each survivor.
Calculate the annual premium paid continuously under this policy assuming this is paid throughout the term or until the second death if death occurs earlier.

Basis:

Mortality: life x , $\mu_x$=0.02 for all x

Life y, $\mu_y$=0.03 for all y

Force of interest: 5% p.a

Expenses: Nil **14marks**



***END***