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MAIN EXAMINATION

SEPTEMBER –DECEMBER 2021

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS

REGULAR PROGRAMME

ACS 402: RISK MATHEMATICS

Date: DECEMBER 2021	**Duration: 2 Hours**
INSTRUCTIONS: Answer Question ONE and any TWO Questions	

Q1.

a) A discrete random variable X has probability generating function given $(0.6 + 0.4t)^n$.

Compute the probability density function of X. **(5 marks)**

b) Cynthia's utility function can be described as U (w) = √w. She faces a potential loss of $50, 000 in the event that her house should burn down, which has a probability of 0.02.

i) Calculate the maximum premium that Cynthia would be prepared to pay to insure herself against the total loss of her house if her initial level of wealth was $100, 000 **(3 marks)**

ii) Suppose that Liberty plc has an initial wealth of $50 million and a utility function of the form U (w) = w, calculate the minimum premium Liberty plc would require in order to offer insurance to Cynthia **(4 marks)**

c) Acontinuous random variable Y has pdf given by the formula g(y) = 0.5e−0.5y for y > 0

i) What is the moment generating function for Y? **(4 marks)**

ii) Compute the variance of Y **(3 marks)**

d) Suppose that an insured party has an exponential utility function with parameter α = 0.0025. Further, suppose that the loss is exponentially distributed with parameter β = 0.02.

i) Compute the value of P +, the maximum premium he is willing to pay for a risk **(4 marks)**

ii) Using the approximation formula, compute the approximated value of **(4 marks)**

e) Derive an expression for the expectation of investor X's next-period wealth if he invests a proportion a of his current wealth w in Equity A (which pays –4% or +8%, with respective probabilities 1 and 3) and the rest in a non-interest-bearing bank account **(3 marks)**

Q2.

a) Investor A has an initial wealth of $100, which is currently invested in a non-interest-bearing account, and a utility function of the form U (w) = log(w) where w is the investor's wealth at any time. Investment Z offers a return of −18% or +20% with equal probability.

i) What is investor A's expected utility if nothing is invested in investment Z? **(3 marks)**

ii) What is investor A's expected utility if they are entirely invested in Investment Z? **(3 marks)**

ii) What proportion a of wealth should be invested in Investment Z to maximize expected utility? **(4 marks)**

iv) What is investor A's expected utility if they invest this proportion in investment Z? **(4 marks)**

b) Suppose investor A has a power utility function with γ = 1 , whilst investor B has a power utility function with γ = 0.5 i) Which investor is more risk-averse (assuming that w > 0)? **(3 marks)**

c) ii) Suppose that investor B has an initial wealth of 100 and is offered the opportunity to buy investment X for 100, which offers an equal chance of a payout of 110 or 92. Will the investor B choose to buy investment X **(3 marks)**

Q3.

a) Suppose that a person owns a capital w and that he values his wealth by the utility function
U (.). He is given the choice of losing the amount b with probability 1 or just paying a fixed amount 1 b. He chooses the former if b = 1, the latter if b = 4, and if b = 2 he is indifferent. Assume that w = 0 and U (0) = 0, U (−1) = −1.
What can be said about the investor's utility function U (.)?

(7 marks)

b) Consider an individual with a utility function of U (x) = $\sqrt{x}$ and current wealth of $15, 000. Assume that this individual is at risk of suffering damages that are uniformly distributed up to 15, 000. Compute the maximum premium that the individual would be willing to pay

(5 marks)

c) An insurer with initial wealth of $2, 000 and a utility of u(x) = log(x) is designing a policy to cover damages of $500 that occur with probability 0.5.
Calculate the minimum premium that the insurer can charge for the policy.

(5 marks)

d) Suppose that an insurer has an exponential utility function with parameter α. What is the minimum premium P− to be asked for a risk X **(3 marks)**

Q4.

a) Suppose that an insured party has an exponential utility function with parameter α.

i) What is the maximum premium P + he is willing to pay for a risk X?

(3 marks)

ii) Suppose that the loss X is exponentially distributed with parameter β = 0.01 and that α = 0.005, what is the value of P +?

(3 marks)

Iii) What is the approximated value of P +?

(3 marks)

b) An investor can invest in two assets, A and B:

	A	B
Expected Return	6%	8%
Variance	4%	25%

The correlation coefficient of the rate of return of the two assets is denoted by ρ and is assumed to take the value 0.5. The investor is assumed to have an expected utility function of the form: $E\alpha(U) = E(rp) - \alpha Var(rp)$ where α is a positive constant and rp is the rate of return on the assets held by the investor

i) Determine, as a function of α , the portfolio that maximizes the investor's expected utility.

(1 marks)

ii) Show that, as α increases, the investor selects an increasing proportion of Asset A**.**

(6 marks)

Q5.

a) What is the probability density of the sum of two independent random variables, each of which is uniformly distributed over the interval [0, 1]? **(4 marks)**

b) Jayne's utility function can be described as $U(w) = \sqrt{w}$. She faces a potential loss of $100, 000 in the event that her should house burn down, which has a probability of 0.01.
i) Calculate the maximum premium that Jayne would be prepared to pay to insure herself against the total loss of her house if her initial level of wealth was $140, 000 and comment on your results (Ans. P + = $1300) **(6 marks)**

ii) Suppose that UN Life plc has an initial wealth of $100 million and a utility function of the form U (w) = w, calculate the minimum premium UN Life plc would require in order to offer insurance to Jayne and comment on whether insurance is feasible in this instance. (Ans. P− = $1000 **(6 marks)**

c) Show that, for quadratic utility, the risk aversion increases with the capital **(4 marks)**

END