THE CATHOLIC UNIVERSITY OF EASTERN AFRICA

A. M. E. C. E. A

P.O. Box 62157

00200 Nairobi - KENYA

Telephone: 891601-6

Ext 1022/23/25

MAIN EXAMINATION

SEPTEMBER –DECEMBER 2021

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS

REGULAR PROGRAMME

ACS 401: SURVIVAL MODELS

Date: DECEMBER 2021	**Duration: 2 Hours**
INSTRUCTIONS: Answer Question ONE and any TWO Questions	

Q1. a) Give the definition of a hazard function **(3 marks)**

b) Find the integrated hazard function of an exponential distribution **(4 marks)**

c) Give the difference between right and left censoring by giving examples **(6 marks)**

d.) Why is informative censoring important in survival analysis? **(6 marks)**

e.) A group of 15 laboratory rats are injected with a new drug. They are observed over the next 30 days. The following events occur:

Day	Event
3	Rat 4 dies from effects of drug.
4	Rat 13 dies from effects of drug.
6	Rat 7 gnaws through bars of cage and escapes.
11	Rats 6 and 9 die from effects of drug.
17	Rat 1 killed by other rats.
21	Rat 10 dies from effects of drug.
24	Rat 8 freed during raid by animal liberation activists.

25 Rat 12 accidentally freed by journalist reporting earlier raid.
26 Rat 5 dies from effects of drug.
30 Investigation closes. Remaining rats hold street party.

How would this information be represented in the notation described above? **(6 marks)**

f.) Butterflies of a certain species have short lives. After hatching, each butterfly experiences a lifetime defined by the following probability distribution:

Lifetime (days)	**Probability**
1	0.10
2	0.30
3	0.25
4	0.20
5	0.15

Calculate $\mu_j for\ j=1,2,3,4$, (to 3 decimal places) and sketch a graph of the discretehazard function. **(4 marks)**

g.) What is the difference between an estimate and an estimator in survival? **(4 marks)**

Q2 a)Derive the form of the likelihood function assuming that the future lifetime random variable follows the Weibull distribution with parameters $\alpha$and β. **(10 marks)**

b.) Give an example of a situation in which the hazard function may be expected to follow each of the following distributions:
(i.) Exponential **(3 marks)**
(ii.) decreasing Weibull **(3 marks)**
(iii.) Gompertz-Makeham **(3 marks)**
(iv.) log-logistic. **(3 marks)**

Q3 The following data relate to 12 patients who had an operation that was intended to correct a life-threatening condition, where time 0 is the start of the period of the investigation:

Patient number	Time of operation	Time observation ended	Reason observation ended
1	0	120	Censored
2	0	68	Death
3	0	40	Death

4	4	120	
Censored			
5	5	35	
Censored			
6	10	40	Death
7	20	120	
Censored			
8	44	115	Death
9	50	90	Death
10	63	98	Death
11	70	120	Death
12	80	110	Death

You can assume that censoring was non-informative with regard to the survival of any individual patient.

(i.) Compute the Nelson-Aalen estimate of the cumulative hazard function, $\gamma(t)$, where t is the time since having the operation. **(12 marks)**

(ii.) Using the results of part (i), deduce an estimate of the survival function for patients who have had this operation. **(4 marks)**

(iii.) Estimate the probability of a patient surviving for at least 70 weeks after undergoing the operation. **(4 marks)**

Q4 A medical study was carried out between 1 January 2001 and 1 January 2006, to assess the survival rates of cancer patients. The patients all underwent surgery during 2001 and then attended 3-monthly check-ups throughout the study.

The following data were collected:

For those patients who died during the study exact dates of death were recorded as follows:

Patient	*Date of surgery*	*Date of death*
A	1 April 2001	1 August 2005
B	1 April 2001	1 October 2001
C	1 May 2001	1 March 2002
D	1 September 2001	1 August 2003
E	1 October 2001	1 August 2002

For those patients who survived to the end of the study:

Patient	*Date of surgery*
F	1 February 2001
G	1 March 2001
H	1 April 2001

I	1 June 2001
J	1 September 2001
K	1 September 2001
L	1 November 2001

For those patients with whom the hospital lost contact before the end of the investigation:

Patient	*Date of surgery*	*Date of last check-up*
M	1 February 2001	1 August 2003
N	1 June 2001	1 March 2002
O	1 September 2001	1 September 2005

(i.) Explain whether and where each of the following types of censoring is present in this investigation:

(a) type I censoring. **(2 marks)**
(b) interval censoring; **(2 marks)**
(c) informative censoring. **(2 marks)**

(ii.) Calculate the Kaplan-Meier estimate of the survival function for these patients. State any assumptions that you make. **(10 marks)**

(iii.) Hence, estimate the probability that a patient will die within 4 years of surgery. **(4 marks)**

Q5 A group of six lives was observed over a period of time as part of a mortality investigation. Each of the lives was under observation at all ages from age 55 until they died or were censored. The table below shows the sex, age at exit and reason for exit from the investigation.

Life	**Sex**	**Age at exit**	**Reason for exit**
1	M	56	death
2	F	62	censored
3	F	63	death
4	M	66	death
5	M	67	censored
6	M	67	censored

The following model has been suggested for the force of mortality:
$\mu(x/Z=z)=\mu_0(x)e^{\beta z}$ where:

- x denotes age

- $\mu_0(x)$ is the baseline hazard
- z = 0 for males and z = 1 for females

Write down the partial likelihood for these observations using the model above.

. **(20 marks)**

END