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MAIN EXAMINATION

SEPTEMBER –DECEMBER 2021

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS

REGULAR PROGRAMME

ACS 202: LINEAR PROGRAMMING
MAT 362: OPERATIONS RESEARCH

Date: DECEMBER 2021	Duration: 2 Hours
INSTRUCTIONS: Answer Question ONE and any TWO Questions	

Q1

(a) Define the following terms as applied in operations Research **(4marks)**
(i) Optimization
(ii) Slack variables
(iii) Artificial variables
(iv) Pivot element

(b) Explain the theorem of duality in linear programming **(2marks)**

(c) A furniture maker has 6 units of wood and 28 hours of free time, in which he will make decorative screens. Two models have sold well in the past, so he will restrict himself to those two. He estimates that model 1 requires 2 units of wood and 7 hours of time while model 2 requires 1 unit of wood and 8 hours of time. The prices of the models are \$120 and \$80 respectively.
How many screens of each model should the furniture make assemble if he wishes to maximize his sales revenue graphically? **(8 marks)**

(d) Set up the dual problem of the following standard minimum problem **(6marks)**

Minimize $z=30x_1+40x_2+50x_3$
Subject to $10x_1+14x_2+5x_3 \geq 220$

$$5x_1+3x_2+9x_3 \geq 340$$
$$x_1, x_2, x_3 \geq 0$$

(e) A firm produces three types of pumps A, B, C, each of which requires the four processes of turning, drilling, assembling and testing.

Pump type	Turning	Drilling	Assembling	Testing	Profit per pump $
A B C	2 1 2	1 1 1	3 4 2	4 3 2	84 72 52
Total available time (h)/week	98	60	145	160	

From the information given in the table, determine

(a) The weekly output of each type of pump to maximize profit using simplex method and

(b) The maximum profit. **(10 marks)**

Q2

(a) Find the graphical solution to the following Linear programming problem **(7marks)**

Maximize $z = x_1 + 4x_2$

Subject to $-x_1 + 2x_2 \leq 6$

$$5x_1 + 4x_2 \leq 40$$
$$x_1, x_2 \geq 0$$

(b) Use simplex method to solve the following linear programming problem

Maximize $z = 8x_1 + 4x_2$

Subject to $2x_1 + 3x_2 \leq 120$

$$x_1 + x_2 \leq 45$$
$$-3x_1 + 5x_2 \geq 25$$

$x_1, x_2 \geq 0$ **(7 marks)**

(c) Use the duality to solve the following linear programming problem **(6 marks)**

Minimize $z = 500x_1 + 700x_2$

Subject to $22x_1 + 30x_2 \geq 110$

$$15x_1 + 40x_2 \geq 95$$
$$20x_1 + 35x_2 \geq 68$$
$$x_1, x_2 \geq 0$$

Q3

(a) A firm makes two types of containers, A and B, each of which requires cutting, assembly and finishing. The maximum available machine capacity in hours per week for each process is: cutting 50, assembly 84, finishing 72. **(10 marks)**

The process times for one unit of each type are as follows

Process	Time in hours	
	A	B
Cutting	2	5
Assembly	4	8
Finishing	4	5

If the profit margin is £600 per unit A and £1000 per unit B, determine

(i) The optimal weekly output of containers

(ii) The maximum profit.

(b) A firm manufactures two types of coupling, A and B, each of which requires processing time on lathes, grinders and polishers. The machine times needed for each type of coupling are given in the table.

Coupling type	Time required (hours)		
	Lathe	Grinder	Polisher
A	2	8	5
B	5	5	2

The total machine time available is 250 hours on lathes, 310 hours on grinders and 160 hours on polishers. The net profit per coupling of type A is £ 9 and of type B £ 10. Determine

(i) The number of each type A units to be produced to maximize the profit.

(ii) The total profit. **(10 marks)**

Q4

(a) Use the simplex method to solve the Linear programming problem **(14 marks)**

Minimize $z = -2x_1 + 8x_2$

Subject to $3x_1 + 4x_2 \leq 80$

$$-3x_1 + 4x_2 \geq 8$$

$$x_1 + 4x_2 \geq 40$$

$$x_1, x_2 \geq 0$$

(b) Put the following Linear programming problem in standard matrix form **(6 marks)**

Minimize $z = 5x_1 + 2x_2$

Subject to $6x_1 + x_2 \geq 6$

$$4x_1 + 3x_2 \geq 12$$

$$x_1 + 2x_2 \geq 4$$

$$x_1, x_2 \geq 0$$

Q5

A company has three manufacturing plants situated at A, B, C with production capacities of 2000, 6000 and 7000 units per week. These units are to be shipped to four distributing Centres D, E, F and G with absorption capacities of 3000, 3000, 4000 and 5000 units per week.

The company would like to distribute the manufactured items in such a way that the total shipping cost from the plants to the distributing Centres is minimum. The various shipping cost per unit product shipped from plants to the Centres are given in the table below.

	To	D	E	F	G
	A	13	11	15	20
From	B	17	14	12	13
	C	18	15	18	12

(a) Formulate the transportation problem

(b) Find the total feasible solution using

(i) North-West corner method

(ii) Least cost method. **(20 marks)**

END