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MAIN EXAMINATION

SEPTEMBER –DECEMBER 2021

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS

REGULAR PROGRAMME

ACS 200: FINANCIAL MATHEMATICS I

Date: DECEMBER 2021	**Duration: 2 Hours**
INSTRUCTIONS: Answer Question ONE and any TWO Questions	

Q1.

a. Express $d^{(4)}$ as a function of $i^{(3)}$. **3marks**

b. Compute the NPV of a perpetuity paying 500 at the end of each year assuming an annual effective interest rate of 3.5%. **4marks**

c. At a certain rate of simple interest, 1,000 will accumulate to 1,100 after a certain period of time. Find the accumulated value of 500 at a rate of simple interest three fourths as great over twice as long a period of time. **4marks**

d. Carl puts 10,000 onto a bank account that pays an annual effective interest rate of 4% for 10 years. If a withdrawal is made during the first five and one-half years, a penalty of 5% of the withdrawal amount is made. Carl withdraws K at the end of each of years 4,5,6,7. The balance in the account at the end of year 10 is 10,000. Calculate K. **6marks**

e. Calculate the nominal rate of interest convertible once every four years that is equivalent to the nominal rate of discount convertible quarterly. **4marks**

f. A constant annual force of interest can be applied over the smallest sub-period imaginable (at a moment in time) and is denoted as δ. Show that $\delta = \ln(1+i)$ **5marks**

g. You buy an increasing perpetuity-due with annual payments starting at 5 and increasing by 5 each year until the payment reaches 100. The payments remains at 100 thereafter. The annual effective interest rate is 7.5%. Determine the present value of this perpetuity. **4marks**

Q2.

a. An investment requires an initial payment of 10,000 and annual payments of 1,000 at the end of each of the first 10 years. Starting at the end of the eleventh year, the investment returns five equal annual payments of *X*. Determine *X* to yield an annual effective rate of 10% over the 15-year period. **5marks**

b. A 10-year investment project requires an initial investment of 1,000,000 and subsequent beginning-of-year payments of 100,000 for the following 9 years. The project is expected to produce 5 annual returns of 600,000 commencing 6 years after the initial investment. Compute the NPV of these cashflows assuming an annual effective interest rate of 5%.

15marks

Q3.

a. A continuous payment stream is received from time 1 to time2. The rate of payment at time t is $\rho(t)=10t+4$. The force of interest is given by $\delta(t)=0.1t+0.04, t\geq 0$. Calculate the present value at time 0 of this payment stream.

10marks

b. Given a nominal interest rate of 7.5% convertible semi-annually, determine the nominal discount rate compounded monthly. **4marks**

c. Maria invests £3,500 at time 0 in order to receive payments of £450 at the end of each of the next 10 years. Determine Maria's effective annual rate of return. **6marks**

Q4.

a. Given that $3a_{\overline{n}|}^{(2)}=2_{\overline{n}|}^{(2)}$ and that $i=3.5\%\,p.a.$ What is the value of n? **5marks**

b. Find the present value of a continuous increasing annuity with a term of 10years if the force of interest is $\delta=0.04$ and if the rate of payment at time t is t^2 per annum. **8marks**

c. A pensioner elects to receive her retirement benefit over 20 years at a rate of 2,000 per month beginning one month from now. The monthly benefit increases by 5% each year. At a nominal interest rate of 6% convertible monthly, calculate the present value of the retirement benefit. **7marks**

Q5.

a. Show that $\ddot{s}_{\overline{n}|}^{(p)}.\,v^n=\ddot{a}_{\overline{n}|}^{(p)}$ **4marks**

b. The present value of a series of payments of 2 at the end of every eight years, forever, is equal to 5. Calculate the effective rate of interest. **6marks**

c. Fund A accumulates at a constant force of interest of $\delta_t^A = \frac{0.05}{1+0.05t}$ *at time* t, for $t \geq 0$, and Fund B accumulates at aconstant force of interest of $\delta^B = 5\%$. You are given:
The amount in Fund A at time zero is 1,000.
The amount in Fund B at time B at time zero is 500.
The amount in Fund C at any time t, for $t \geq 0$, is equal to the sum of the amount in Fund A and Fund B. Fund C accumulates at a force of interest of δ_t^C, for $t = 0$. Calculate δ_t^C.

10marks

END